

Relationships Between two Port Parameters

① Relationship between Z-Parameter & Y-Parameter.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

- This two equation can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} I \end{bmatrix} \rightarrow \textcircled{1}$$

and Y-Parameter

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix} \begin{bmatrix} V \end{bmatrix} \rightarrow \textcircled{2}$$

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بالقوة $\textcircled{1}$ و $\textcircled{2}$

$$\therefore \cancel{[V]} = \cancel{[Z]} \cancel{[Y]} \cancel{[V]}$$

$$[Z][Y] = 1 \rightarrow (3)$$

From (3) $[Z] = [Y]^{-1}$

inverse ال عكس

$$[Y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$Y^T = \begin{bmatrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{bmatrix}$$

$$Y^a = \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta Y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix} = y_{11} y_{22} - y_{12} y_{21}$$

$$\therefore [Y^{-1}] = \frac{1}{\Delta Y} \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix} = \begin{bmatrix} \frac{y_{22}}{\Delta Y} & \frac{-y_{12}}{\Delta Y} \\ -\frac{y_{21}}{\Delta Y} & \frac{y_{11}}{\Delta Y} \end{bmatrix}$$

$$\therefore \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{y_{22}}{\Delta Y} & \frac{-y_{12}}{\Delta Y} \\ -\frac{y_{21}}{\Delta Y} & \frac{y_{11}}{\Delta Y} \end{bmatrix}$$

$$z_{11} = \frac{y_{22}}{\Delta Y} \quad z_{12} = \frac{-y_{12}}{\Delta Y} \quad z_{21} = \frac{-y_{21}}{\Delta Y} \quad z_{22} = \frac{y_{11}}{\Delta Y} \quad \text{X}$$

Relationship between Y-Parameter and Z-Parameter.

→ From equation 3 $[Y] = [Z]^{-1}$

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z^T = \begin{bmatrix} Z_{22} & Z_{21} \\ Z_{12} & Z_{11} \end{bmatrix}$$

$$Z^a = \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\therefore [Z]^{-1} = \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \quad \& \quad Y_{12} = -\frac{Z_{12}}{\Delta Z} \quad \& \quad Y_{21} = -\frac{Z_{21}}{\Delta Z} \quad \& \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Relation between h-Parameter and Z-Parameter and y-Parameter

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

From (1) $h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0}$ $V_2 = 0$ short circuit

$\Rightarrow$ in Y-Parameter equations.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \Rightarrow \left. \frac{I_1}{V_1} \right|_{V_2=0} = Y_{11}$$

$$\therefore \left. \frac{V_1}{I_1} \right|_{V_2=0} = \frac{1}{Y_{11}} = h_{11}$$

$$\therefore h_{11} = \frac{1}{Y_{11}} = \frac{\Delta Z}{Z_{22}} \quad \text{**}$$

$$\Rightarrow h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \quad \text{open circuit}$$

From Z-Parameter's equations

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\stackrel{I_1=0}{\Rightarrow} V_1 = Z_{12} I_2$$

$$\Rightarrow V_2 = Z_{22} I_2$$

$\therefore$ divided two equations $h_{12} = \frac{V_1}{V_2} = \frac{Z_{12}}{Z_{22}}$

$$\therefore h_{12} = \frac{V_1}{V_2} = \frac{Z_{12}}{Z_{22}}$$

$$h_{12} = \frac{-Y_{12}}{\Delta Y} \times \frac{\Delta Y}{Y_{22}} = \frac{-Y_{12}}{Y_{22}} \quad \therefore h_{12} = \frac{-Y_{12}}{Y_{22}} \quad \text{**}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} \quad V_2=0 \quad \text{Short circuit}$$

From y-Parameter equation

$$\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 & V_2=0 &\Rightarrow I_1 = y_{11} V_1 \\ I_2 &= y_{21} V_1 + y_{22} V_2 & &\Rightarrow I_2 = y_{21} V_1 \end{aligned}$$

$$\frac{I_2}{I_1} \Big|_{V_2=0} = \frac{y_{21}}{y_{11}} = h_{21}$$

$$\therefore \boxed{h_{21} = \frac{y_{21}}{y_{11}}}$$

$$h_{21} = -\frac{Z_{21}}{\Delta Z} \times \frac{\Delta Z}{Z_{22}} = -\frac{Z_{21}}{Z_{22}}$$

$$\boxed{h_{21} = -\frac{Z_{21}}{Z_{22}}}$$

$$\Rightarrow h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} \quad I_1=0 \quad \text{Open circuit}$$

From Z-Parameter equations

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad I_1=0 \Rightarrow V_2 = Z_{22} I_2$$

$$\therefore \frac{I_2}{V_2} \Big|_{I_1=0} = \frac{1}{Z_{22}} = h_{22}$$

$$\boxed{h_{22} = \frac{1}{Z_{22}}}$$

$$\boxed{h_{22} = \frac{\Delta y}{y_{11}}}$$

- Relation between Z-Parameter and h-Parameters.

h-Parameter

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Z-Parameter

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\rightarrow Z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0}$$

بالقوة في مداخل $I_2=0$ مع h

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$0 = h_{21} I_1 + h_{22} V_2 \Rightarrow$$

$$\boxed{V_2 = -\frac{h_{21}}{h_{22}} I_1}$$

بالقوة!

$$V_1 = h_{11} I_1 + h_{12} \left(-\frac{h_{21}}{h_{22}}\right) I_1$$

$$V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}} \right] I_1$$

$$V_1/I_1 = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}$$

$$\Delta h = h_{11} h_{22} - h_{12} h_{21}$$

$$\therefore \boxed{Z_{11} = \frac{\Delta h}{h_{22}}}$$

$$\Rightarrow Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

بالقوف في مخرج h -Parameter

$$\begin{aligned} I_2 &= h_{21} I_1 + h_{22} V_2 & \xrightarrow{I_1=0} & I_2 = h_{22} V_2 \\ V_1 &= h_{11} I_1 + h_{12} V_2 & \Rightarrow & V_1 = h_{12} V_2 \end{aligned}$$

$$\therefore \boxed{\frac{V_1}{I_2} = \frac{h_{12}}{h_{22}} = Z_{12}}$$

$$\therefore \boxed{Z_{12} = \frac{h_{12}}{h_{22}}}$$

$$\Rightarrow Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

بالقوف في مخرج h -Parameter مع $I_2=0$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\hookrightarrow 0 = h_{21} I_1 + h_{22} V_2$$

$$\therefore \frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}$$

$$\therefore \boxed{Z_{21} = -\frac{h_{21}}{h_{22}}}$$

$$\Rightarrow Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

بالقوف في مخرج h -par. مع $I_1=0$

$$I_2 = h_{22} V_2$$

$$\therefore \frac{V_2}{I_2} = \frac{1}{h_{22}}$$

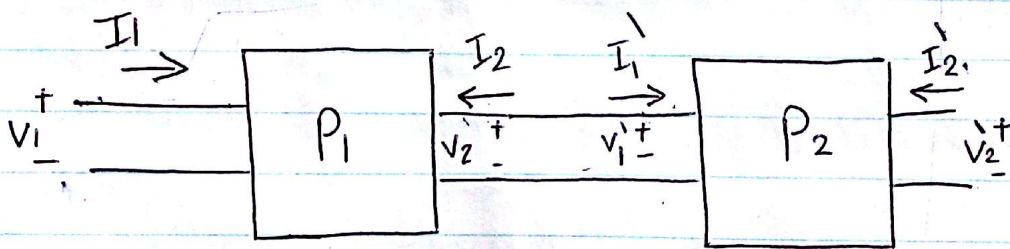
$$\therefore \boxed{Z_{22} = \frac{1}{h_{22}}}$$

Interconnection of two - ports :-

هناك طرق عدة لتوصيل دائرتين معا "متجاورتين"

- 1- Cascade.
- 2- Series
- 3- Parallel
- 4- Series - Parallel
- 5- Parallel - series

① Cascade :-

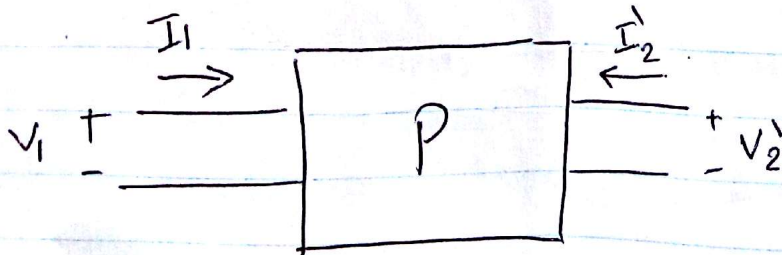


$$V_2 = V_1'$$

$$I_2 = -I_1'$$

نلاحظ من الرسم

نلاحظ من الرسم المكافئ لهذه الدائرة



→ For P_1 :- write the Transmission

$$V_1 = A \cdot V_2 - B \cdot I_2 \quad \text{OR} \quad V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = C \cdot V_2 - d \cdot I_2 \quad \text{OR} \quad I_1 = a_{21} V_2 - a_{22} I_2$$

For P_1

$$V_1 = a_{11} V_2 - a_{12} I_2$$

$$I_1 = a_{21} V_2 - a_{22} I_2$$

For P_2

$$V_1' = a_{11}' V_2' - a_{12}' I_2'$$

$$I_1' = a_{21}' V_2' - a_{22}' I_2'$$

Since $V_1' = V_2$ & $I_2 = -I_1'$

بالعكس المعادله

$$V_1 = a_{11} (a_{11}' V_2' - a_{12}' I_2') + a_{12} (a_{21}' V_2' - a_{22}' I_2')$$

$$= a_{11} a_{11}' V_2' - a_{11} a_{12}' I_2' + a_{12} a_{21}' V_2' - a_{12} a_{22}' I_2'$$

$$V_1 = (a_{11} a_{11}' + a_{12} a_{21}') V_2' + (-a_{11} a_{12}' - a_{12} a_{22}') I_2'$$

$$V_1 = (a_{11} a_{11}' + a_{12} a_{21}') V_2' - (a_{11} a_{12}' + a_{12} a_{22}') I_2' \rightarrow \textcircled{1}$$

and I_1

$$I_1 = a_{21} (a_{11}' V_2' - a_{12}' I_2') + a_{22} (a_{21}' V_2' - a_{22}' I_2')$$

$$I_1 = a_{21} a_{11}' V_2' - a_{21} a_{12}' I_2' + a_{22} a_{21}' V_2' - a_{22} a_{22}' I_2'$$

$$I_1 = (a_{21} a_{11}' + a_{22} a_{21}') V_2' - I_2' (a_{21} a_{12}' + a_{22} a_{22}')$$

$\textcircled{2}$

From (1) & (2)

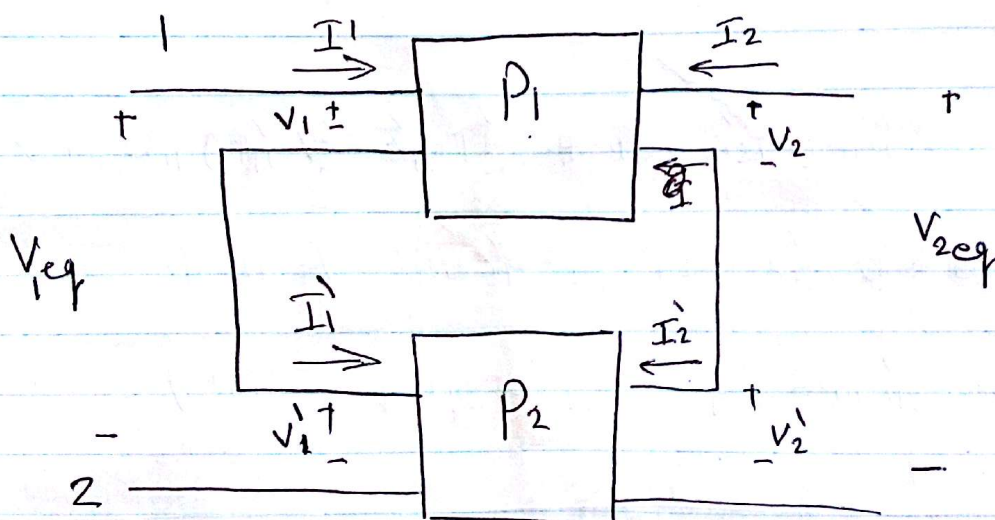
$$a_{11eq} = a_{11} \dot{a}_{11} + a_{12} \dot{a}_{21}$$

$$a_{12eq} = a_{11} \dot{a}_{12} + a_{12} \dot{a}_{22}$$

$$a_{21eq} = a_{21} \dot{a}_{11} + a_{22} \dot{a}_{21}$$

$$a_{22eq} = a_{21} \dot{a}_{12} + a_{22} \dot{a}_{22}$$

② Series



$$I_1 = I_1' \text{ \& } I_2 = I_2' \quad \text{سریع ہر ایک}$$

$$V_{1eq} = V_1 + V_1'$$

$$V_{2eq} = V_2 + V_2'$$

For P_1 :

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

For P_2

$$V_1' = Z_{11}' I_1' + Z_{12}' I_2'$$

$$V_2' = Z_{21}' I_1' + Z_{22}' I_2'$$

Then

$$V_{eq} = Z_{11} I_1 + Z_{12} I_2 + Z_{11}' I_1' + Z_{12}' I_2'$$

$$I_1' = I_1, \quad I_2' = I_2$$

$$\therefore V_{eq} = (Z_{11} + Z_{11}') I_1 + (Z_{12} + Z_{12}') I_2$$

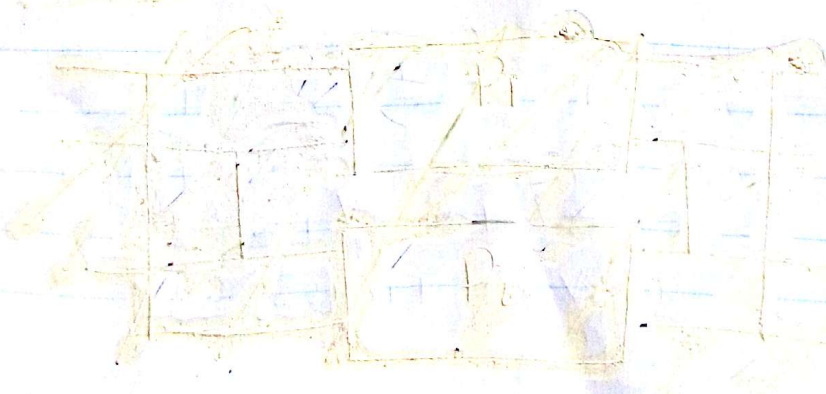
$$V_2 eq = Z_{21} I_1 + Z_{22} I_2 + Z_{21}' I_1' + Z_{22}' I_2'$$

$$V_2 eq = (Z_{21} + Z_{21}') I_1 + (Z_{22} + Z_{22}') I_2$$

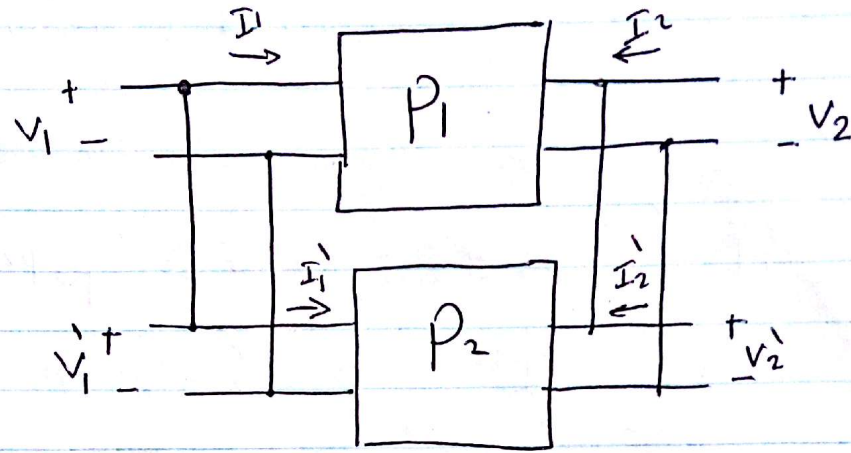
$$Z_{11eq} = Z_{11} + Z_{11}', \quad Z_{12eq} = Z_{12} + Z_{12}'$$

$$Z_{21eq} = Z_{21} + Z_{21}', \quad Z_{22eq} = Z_{22} + Z_{22}'$$

Parallel



③ Parallel:-



$$V_1 = V_1' \quad \text{and} \quad V_2 = V_2'$$

$$I_{1eq} = I_1 + I_1' \quad , \quad I_{2eq} = I_2 + I_2'$$

$$Y_{11eq} = Y_{11} + Y_{11}' \quad \text{and} \quad Y_{12eq} = Y_{12} + Y_{12}'$$

$$Y_{21eq} = Y_{21} + Y_{21}' \quad \text{and} \quad Y_{22} = Y_{22} + Y_{22}'$$